

## PART A

## ANSWER ALL QUESTIONS:

1. What are constraints? Classify the constraints with examples.
2. If $F=\left(2 x y+z^{2}\right) i+x^{2} j+2 x z k N$, then show that is a conservative force
3. Explain the terms products of inertia and moment of inertia coefficients.
4. Calculate $\mathrm{I}_{\mathrm{xy}}, \mathrm{I}_{\mathrm{xz}}$ for the system of four point masses $1 \mathrm{~g}, 2 \mathrm{~g}, 3 \mathrm{~g}$ and 4 g located at the points $(1,0,0)$, $(1,1,0),(1,1,1)$ and $(1,1,-1)$
5. Show that $\left[p_{x}, L_{z}\right]=-p_{y}$
6. Show that the function $\mathrm{F}=-\sum$ Qi pi generates the identity transformation
7. Express the Hamiltonian for a particle in a central potential $\mathrm{V}(\mathrm{r})$ using polar coordinates in terms of the Hamilton's characteristic function W.
8. The Lagrangian for anharmonic oscillator is given by $L=\frac{1}{2} \dot{x}^{2}-\frac{\omega^{2} x^{2}}{2}-\alpha x^{3}$. Find the Hamiltonian.
9. Explain the terms stable and unstable equilibrium.
10. What are coupled oscillators?

## PART B

## ANSWER ANY FOUR QUESTIONS:

11. Obtain the equation of motion of a simple pendulum by using Lagrangian method and hence deduce the formula for its time period for small amplitude oscillations
12. Establish the relation between inertia tensor and angular momentum vector.
13. Using the variational principle show that the shortest distance between two points in a plane is a straight line.
14. What are action -angle variables? Determine the frequency of a harmonic oscillator using actionangle variables.
15. Deduce the eigenvalue equation for small oscillations.
16. Prove that $\mathrm{P}=\mathrm{q} \cot \mathrm{p}$ and $\mathrm{Q}=\log \left(\frac{\sin \mathrm{p}}{\mathrm{q}}\right)$ is canonical and find the generating function.

## PART C

## ANSWER ANY FOUR QUESTIONS:

$(4 \times 12.5=50)$
17. State and prove the Kepler's laws of planetary motion.
18. Define Euler"s angles and obtain an expression for the complete transformation matrix.
19. Define canonical transformations and obtain the transformation equations corresponding to all possible generating functions.
20. Prove by Hamilton Jacobi theory that the orbit of a planet around the sun is an ellipse.
21. Determine the normal mode frequencies of a double pendulum
22. For a system with Lagrangian $\mathrm{L}=\frac{1}{2}\left(\mathrm{q}_{1}^{2}+\mathrm{q}_{1} \dot{q}_{2}+\mathrm{q}_{2}^{2}\right)-V(q)$, show that the Hamiltonian is $\mathrm{H}=$ $\frac{2}{3}\left(p_{1}^{2}-p_{1} p_{2}+p_{2}^{2}\right)+V(q)$.

